A Combination of FD-TD and Prony's Methods for Analyzing Microwave Integrated Circuits

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Abstract—**It is demonstrated in this paper that in applying the FD-TD technique to analyze microwave integrated circuits, the long FD-TD time record required for generating accurate frequency domain scattering parameters can be extrapolated from a relatively short FD-TD time record by using Prony's method. As shown by comparison with the direct FD-TD generated results, the new approach using the combination of FD-TD and Prony's methods achieves the same type of accuracy with a time record computed over a much shorter time.**

I. Introduction

IN APPLYING the FD-TD technique [1], [2] to analyze microwave integrated circuits, the transient time record may be required over tens of thousands of time steps in order to obtain accurate frequency domain scattering parameters via Fourier transformation. Depending on the complexity of the microwave integrated circuit being analyzed, which dictates the size of the spatial sampling grid to be used in modeling the physical structure, the computational time can run up to several hours. As shown in this paper, the accuracy of the scattering parameters is compromised if the transient time record is terminated too prematurely. However, it is also demonstrated in this paper that the required FD-TD time response can be efficiently obtained from a relatively short time record by using an extrapolation scheme based upon the Prony's method [3], [4].

II. Method of Approach

A combination of the FD-TD and Prony's methods is used to analyze microwave integrated circuits. The FD-TD technique is summarized as follows.

1) Fill computational space as shown in Fig. 1 with Yee's cells.
2) Truncate the computational space with reflectionless walls [5].
3) Place structure in the computational volume.

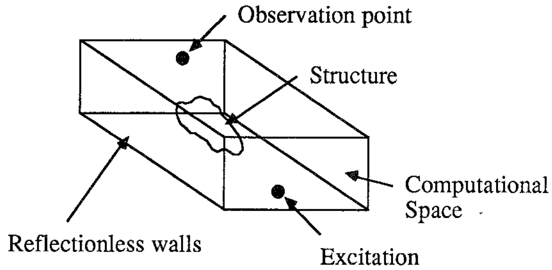

Fig. 1. FD-TD computational space.

4) Excite the structure with a Gaussian pulse whose width is chosen to cover the frequency bandwidth of interest.
5) Observe the transient wave form in the time-domain at a proper location.
6) Extract the scattering parameter in the frequency-domain by using Fourier transform.

As a first step in the extrapolation scheme, a curve fitting procedure is employed to approximate the time record, computed over a relatively short period using the conventional FD-TD scheme, in terms of complex exponential functions whose frequencies of oscillation and damping factors are determined by Prony's method. In this method, the late time transient wave form is approximated by a sum of damped complex exponentials in the following manner:

$$g_n = \sum_{k=1}^{K} c_k z_k^n, \qquad n = 1, 2, \cdots, N \tag{1}$$

where c_k and z_k are complex quantities which can be expressed as follows:

$$c_k = A_k \exp(j\phi_k) \tag{2}$$

and

$$z_k = \exp[(-\alpha_k + j2\pi f_k)\Delta t] \tag{3}$$

where

A_k is the amplitude,
ϕ_k is the phase,
α_k is the damping factor,
f_k is the frequency of the kth resonance,
Δt is the sampling interval.

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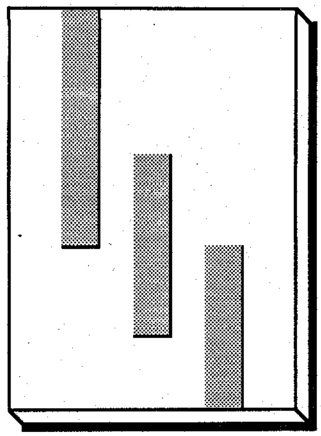

Fig. 2. Band-pass filter.

It is readily verified that g_n satisfies a difference equation

$$g_n = -\sum_{k=1}^{K} b_k g_{n-k}, \qquad n = K+1,\cdots,N \tag{4}$$

where $b_k, k = 1,\cdots,K$ are the coefficients of the polynomial

$$P(z) = z^K + b_1 z^{K-1} + \cdots + b_K \tag{5}$$

which has complex roots $z_1,\cdots,z_K$ for representing g_n as given in (1).

In (4), $g_n, g_{n-1}, g_{n-2},\cdots, g_{n-K}$ are known quantities obtained from sampling the FD-TD time record. The coefficients b_k can be solved for as the least-squares solution to the over determined system of linear equations in (4). Having determined the b_k's, we can proceed to find the roots of $P(z)$ in (5) since the coefficients of that polynomial now become known. These roots are estimates of the damping factors and frequencies as shown in (3). The final step involves substituting these roots, $z_k, k = 1,2,\cdots,K$, into (1) and solving the resulting least squares problem to estimate the c_k, $k = 1,2,\cdots,K$. As shown in (2), these c_k's are complex quantities with amplitudes A_k and phases ϕ_k.

The results for the future time steps, i.e., the late time FD-TD record, can now be generated very efficiently using the representation given in (1) in terms of complex exponential while bypassing the time-consuming FD-TD computation altogether. The time saving resulting from the use of this extrapolation scheme can be very substantial indeed and it is not unusual to realize a reduction in computation time from hours to seconds.

III. Numerical Results

The usefulness of the scheme outlined above is illustrated in this section by considering a microstrip band-pass filter shown in Fig. 2, which is known to require a very long FD-TD time record of over 30 000 iterations (See Fig. 3), in order to obtain accurate S-parameters depicted in Fig. 4, that compare well with the experimental data in [6].

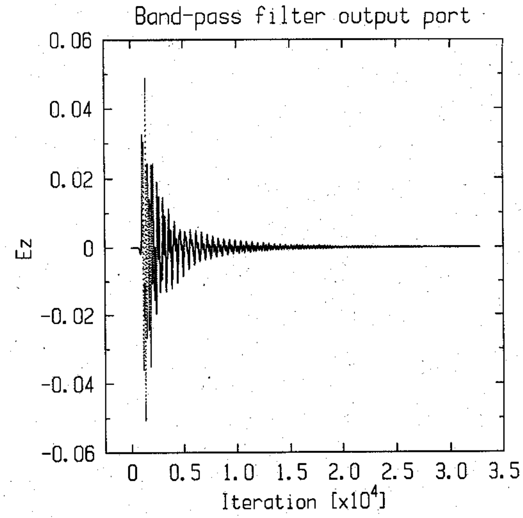

Fig. 3. FD-TD time record of 33 000 iterations.

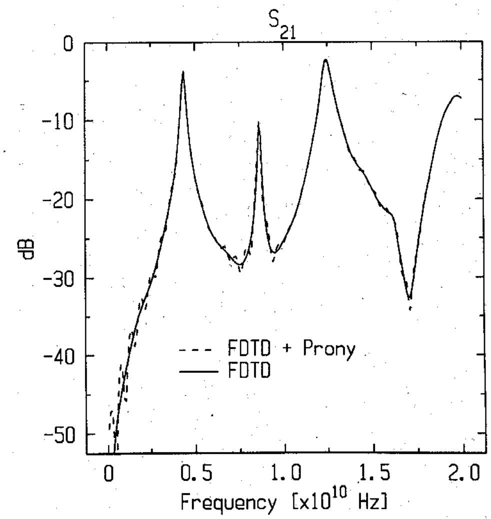

Fig. 4. S-parameter obtained by Fourier transformation of Fig. 3.

Using the new approach, a relatively short FD-TD time record of 5000 iterations is generated as shown in Fig. 5. A window of 2000 iterations is sampled at every 20 iterations to obtain 100 data points from which complex conjugate pairs of resonance frequencies and coupling coefficients are found by applying Prony's method. After experimenting a few times by shifting the window up and down the FD-TD time record, it is found that an 18-term series of complex exponential functions fits the FD-TD data satisfactorily as shown in Fig. 6. Using this 18-term series, a good approximation of the long FD-TD record of 30 000 iterations can be computed in seconds, saving hours of iteration time in the process.

Fig. 7 shows the comparison between the waveform extrapolated by the new approach and the direct FD-TD waveform at late time, e.g., between the 7000 and 10 000 iterations. If the truncated time record in Fig. 5 is used to compute the S-parameter, and the result is compared with the S-parameter generated by the present approach, as illustrated in Fig. 8, one sees that the peaks in the

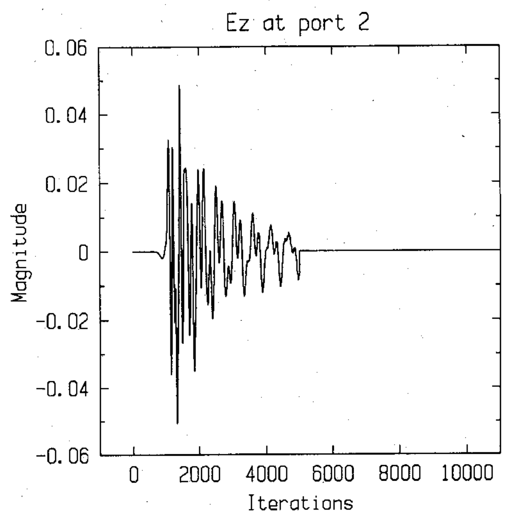

Fig. 5. FD-TD time record of 5000 iterations.

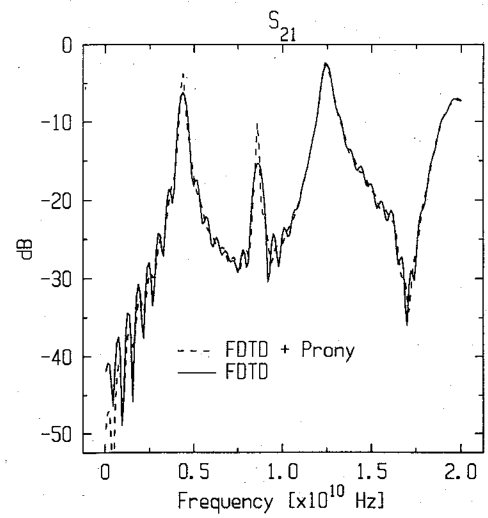

Fig. 8. Sharpening of the peaks in the S-parameter computation by including the late time transient waveform.

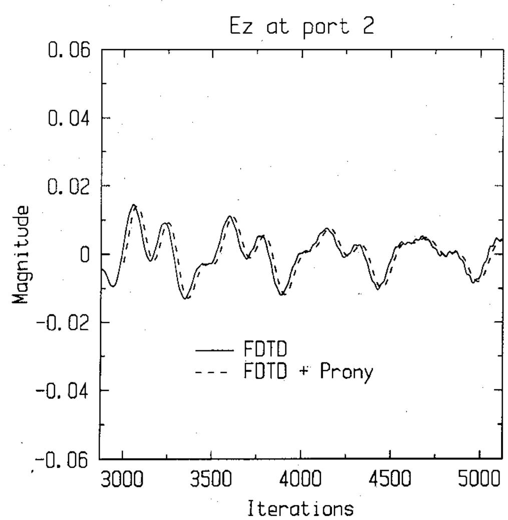

Fig. 6. Curve-fitting by Prony's method in a window of 2000 iterations.

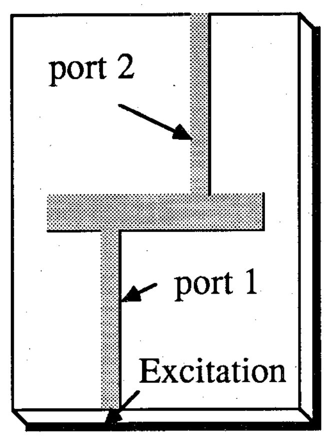

Fig. 9. Low pass filter.

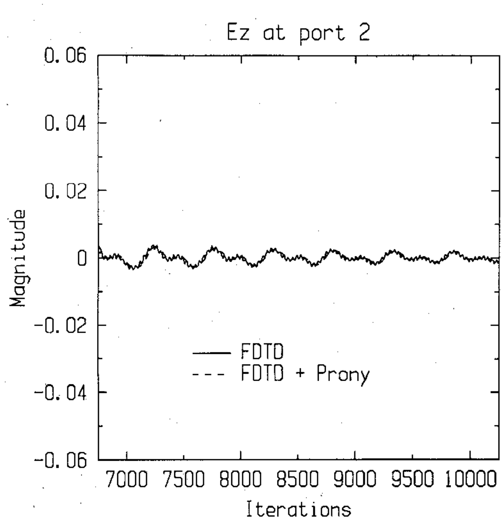

Fig. 7. Extrapolating the time transient waveform to late time.

S-parameter have been sharpened by including the late time transient waveform in the Fourier transform computation.

Having demonstrated the essential steps in applying the combination of FD-TD and Prony's methods to analyze the bandpass filter, we now validate the technique by applying it to analyze various structures in [7], which is chosen because experimental data are also available for comparison with the computational results. Numerical results obtained by the present approach for the lowpass filter in Fig. 9 are presented in Figs. 10 to 14, which are in good agreement with those measured as well as computed by direct FD-TD in [7]. Fig. 15 emphasizes that the same degree of accuracy in computing the frequency response in the pass band of the low pass filter is achieved by the present approach. The accuracy of the computation of the frequency response is compromised if the transient time record is terminated prematurely as shown in Fig. 16.

The actual computation time reduction realized in the implementation of the present approach to analyze microwave integrated circuits depends on the structures under investigation. The CPU times for the bandpass

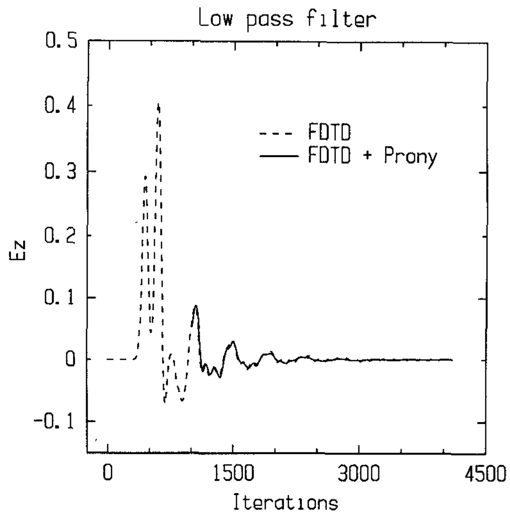

Fig. 10. Comparison of a direct 4096 iteration FD-TD time transient waveform at the output port of the low pass filter and the composite waveform generated by curve fitting a 2000 iteration FD-TD time record in a window of 1000 iterations from 1000 to 2000 iterations and extrapolating it to 4096 time steps.

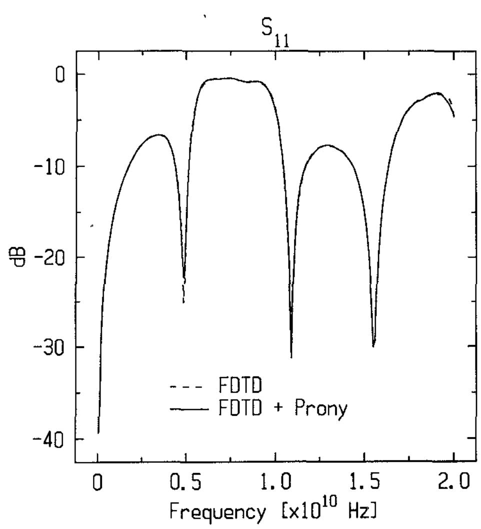

Fig. 11. Scattering parameter S_{11} of the low pass filter obtained by using a 4096 iteration FD-TD time record and a 4096 iteration time waveform extrapolated from a 2000 iteration FD-TD time record.

filter analyzed are listed in Table I and those for the low pass filter are in Table II. Generally speaking, a more significant reduction in computation time is realized for a structure that requires a longer FD-TD time record to generate accurate scattering parameters in the frequency domain.

The window, which is that portion of the short FD-TD time record sampled for curve fitting by Prony's method, is usually at the end of the short FD-TD time record. This window covers a significant fraction of the short FD-TD time record but does not include the beginning of the FD-TD waveform. The FD-TD waveform within the window is a steadily decaying oscillation. The FD-TD waveform is sampled frequently enough to allow effective curve fitting. The success of the curve fitting is checked

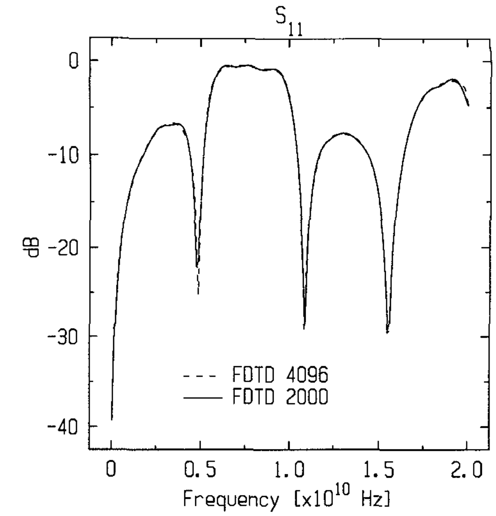

Fig. 12. Scattering parameter S_{11} of the low pass filter obtained by using a 4096- and a 2000-iteration FD-TD time record.

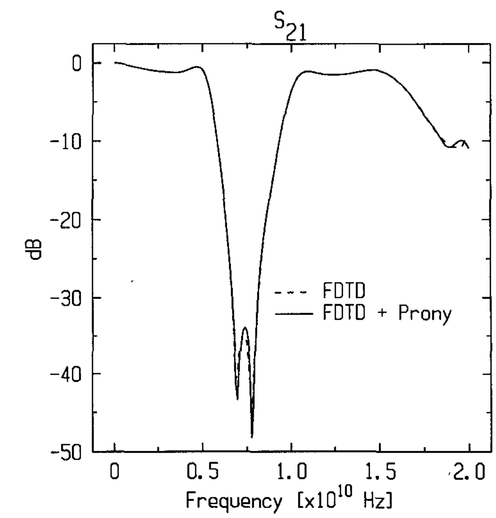

Fig. 13. Scattering parameter S_{21} of the low pass filter obtained by using a 4096 iteration FD-TD time record and a 4096 iteration time waveform extrapolated from a 2000 iteration FD-TD time record.

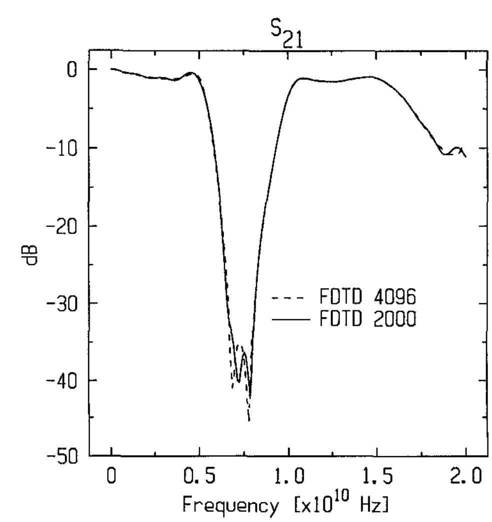

Fig. 14. Scattering parameter S_{21} of the low pass filter obtained by using a 4096- and a 2000-iteration FD-TD time record.

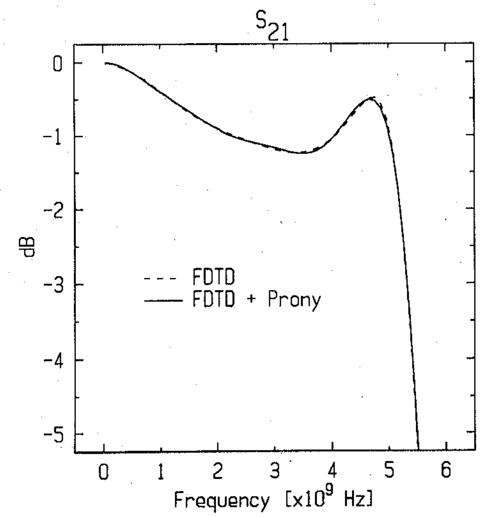

Fig. 15. Frequency response in the pass band of the low pass filter obtained by using a 4096 iteration FD-TD time record and a 4096 iteration time waveform extrapolated from a 2000 iteration FD-TD time record.

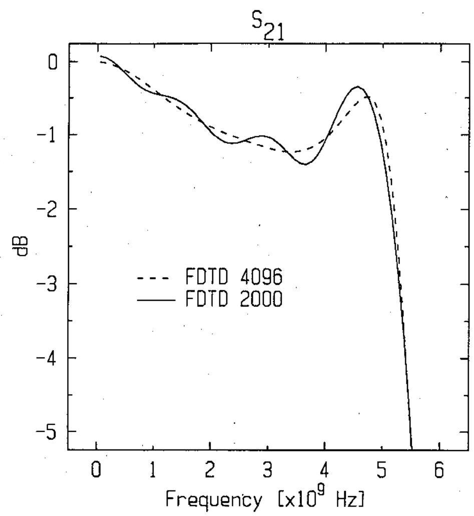

Fig. 16. Frequency response in the pass band of the low pass filter obtained by using a 4096- and a 2000-iteration FD-TD time record.

virtually as shown in Fig. 6 between the 3000 and 5000 iterations or in Fig. 10 between the 1000 and 2000 iterations. If these guidelines for choosing the window for curve fitting are followed, a relatively small number of terms in Prony's method, e.g., 18 terms for Figs. 6 and 7, and 20 terms for Fig. 10, will result in a satisfactory curve fitting and subsequently a successful extrapolation.

IV. Conclusion

As shown by comparison with the direct FD-TD generated results, the new approach using the combination of FD-TD and Prony's methods achieves the same degree of accuracy in a much shorter time. This method can be used for other potential applications that require the computation of response waveforms over a long period of time in order to obtain an accurate frequency domain result derived by Fourier transformation.

TABLE I
CPU Times for Bandpass Filter

5000 FD-TD iterations using $100\times 80\times 16$ Yee's cells on a computer rated at 64 MFLOP	1.5278 h
Prony's extrapolation to 35 000 time steps, including 5 approximations to obtain 18 terms to fit the 100 points obtained by sampling at every 20 iterations in a window of 2000 FD-TD iterations between the 3000 and 5000 iterations.	74.88 s
Additional 30 000 FD-TD iterations to 35 000 time steps	9.1667 h

TABLE II
CPU Times for Low Pass Filter

2000 FD-TD iterations using $100\times 80\times 16$ Yee's cells on a computer rated at 64 MFLOP	0.6112 h
Prony's extrapolation to 4096 time steps, including 6 approximations to obtain 20 terms to fit the 100 points obtained by sampling at every 10 iterations in a window of 1000 FD-TD iterations between the 1000 and 2000 iterations.	87.36 s
Additional 2096 FD-TD iterations to 4096 time steps	0.6404 h

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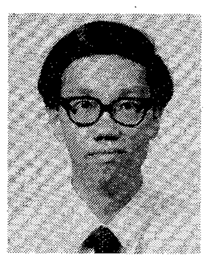

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